Math 131A-3: Homework 8

Due: November 25, 2013

1. Do problems 29.5, 29.13, 29.16, 29.18, 23.1(a),(c),(e),(g), 23.5[You are welcome to cite Theorem 12.1], 31.1, 31.4 in Ross.

2. *The five constants.* Recall that the imaginary number i satisfies the property that $i^2 = -1$. Assume that the power series expansions about zero we have computed for e^x, $\sin x$, and $\cos x$ are valid on complex numbers as well as real numbers. (This is true, but we won't prove it in this class.)

   - (a) What are i^3 and i^4? In general, what can you say about i^{4k+j}?
   - (b) Use the power series expansions for e^x, $\sin x$, and $\cos x$ to show that $e^{ix} = \cos x + i \sin x$ for all $x \in \mathbb{R}$.
   - (c) Put $x = \pi$ into the equation from part to prove that $e^{i\pi} + 1 = 0$. This gives a relationship between our five most basic analytical constants.